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Title: High-Order Discontinuous Galerkin Schemes with Globally Divergence-Free Constrained Transport for ideal MHD

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The modification of the celebrated Yee scheme from the vacuum Maxwell equations to magnetohydrodynamics (MHD) is often referred to as the constrained transport (CT) approach. Constrained transport can be viewed as a sort of predictor-corrector method for updating the magnetic field, where a magnetic field value is first predicted by a method that does not exactly preserve the divergence-free condition on the magnetic field, followed by a correction step that aims to control these divergence errors. This strategy has been successfully used in conjunction with a variety of shock-capturing methods including WENO (weighted essentially non-oscillatory), central, and wave propagation schemes. In this work we show how to extend the basic CT framework in the context of the discontinuous Galerkin (DG) finite element method on both Cartesian and triangle-based unstructured grids. We first review the entropy-stability theory for semi-discrete DG discretizations of ideal MHD, which rigorously establishes the need for both local and global divergence-free finite element representations of the magnetic field. Next we introduce a novel constrained transport scheme that is based on achieving a globally divergence-free magnetic field via the use of a magnetic potential. We develop schemes both for Cartesian (i.e., structured) and triangular (i.e., unstructured) meshes. The resulting methods are validated on several standard test cases for ideal MHD.